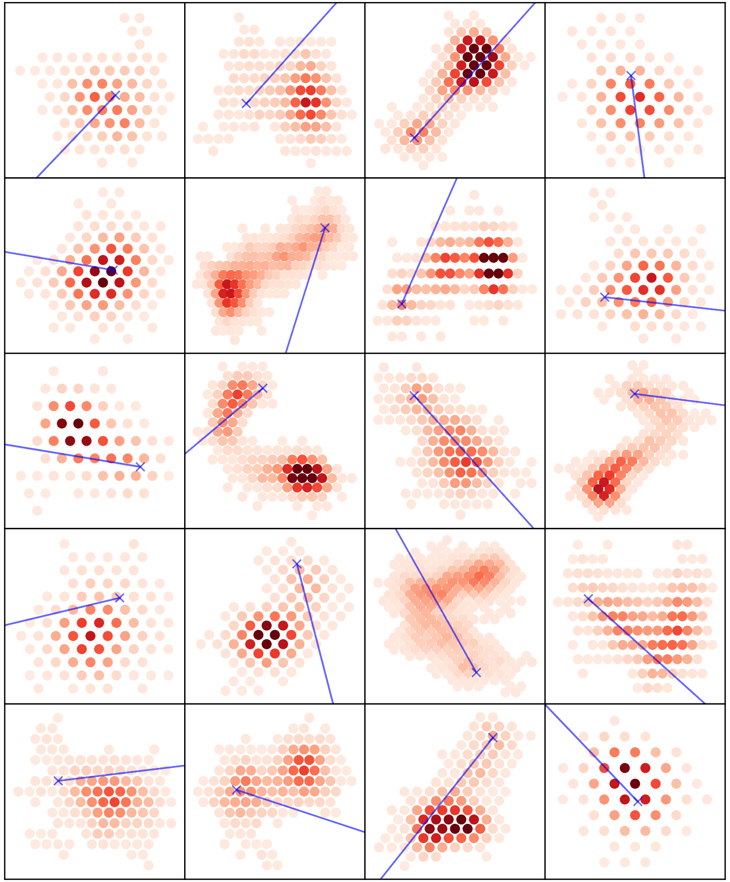


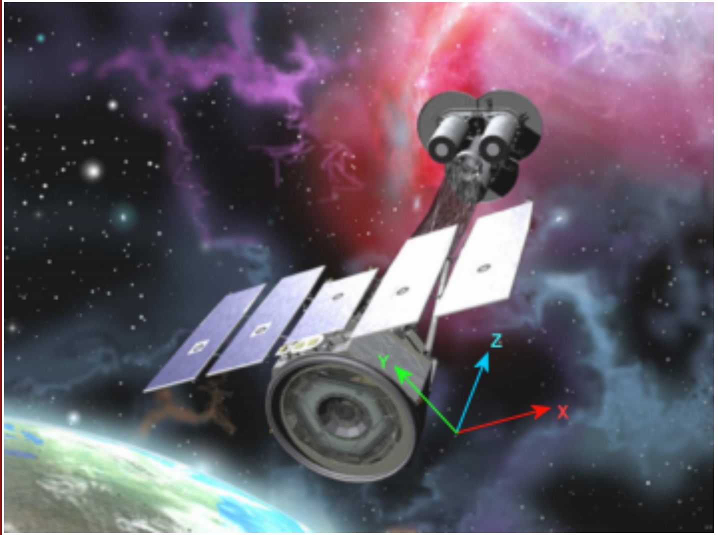
Optimal Signal Extraction for IXPE with an Application to Blazars

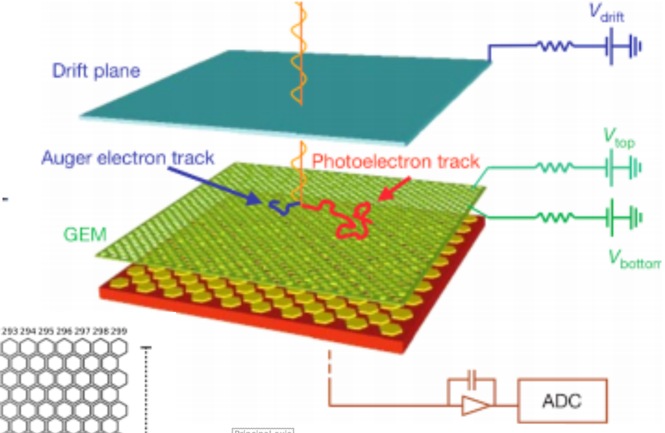
A. Lawrence Peirson

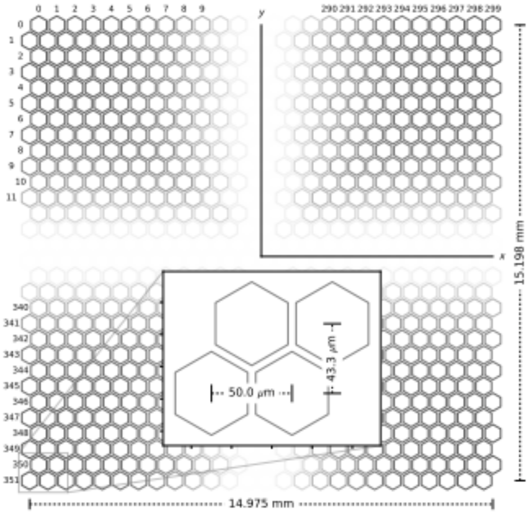
23/09/21

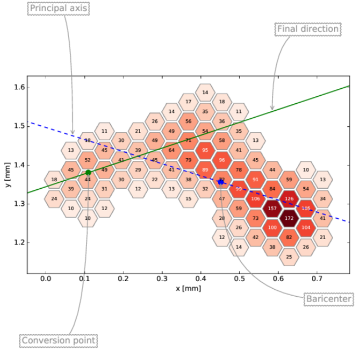
Based on Peirson & Romani ApJ 2021, ApJ 2020, ApJ 2019

*No earlier than December 13th 2021









Baldini et al. 2021 Astroparticle Physics, 133, 102628

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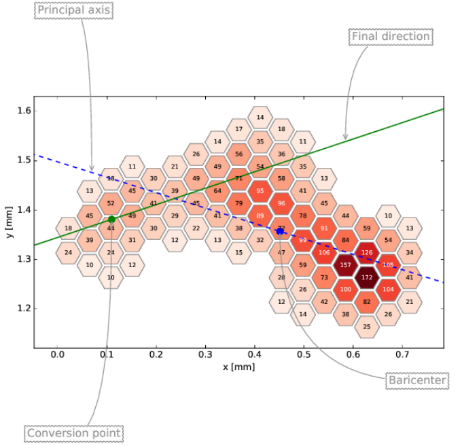
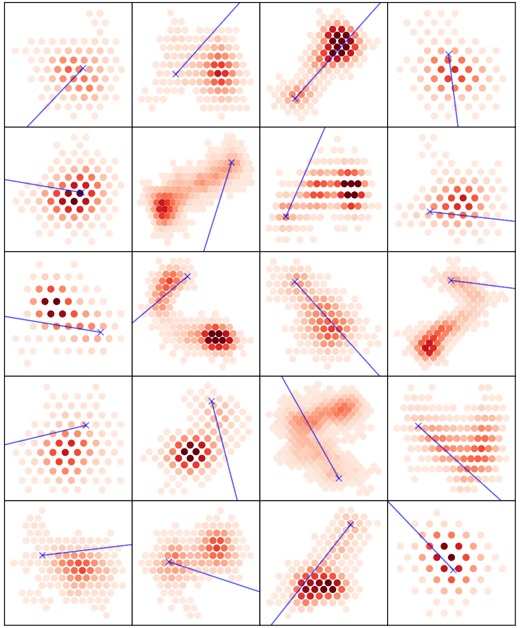
A Difficult Statistical Problem

Step 1



Individual event:
Photoelectron Track

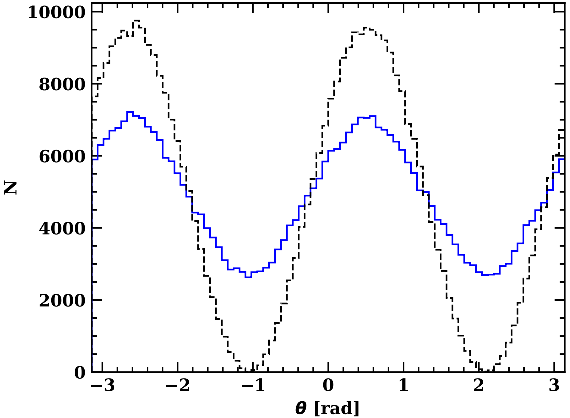
$$\theta, x, y, E$$



Ensemble of events

$$\{\theta_i, x_i, y_i, E_i\}_{i=1}^{N}$$

Step 2



$$\theta \sim \frac{1}{2\pi}\left(1 + \boxed{p_0}\,\mu_{100}\cos[2(\theta - \boxed{\theta_0})]\right)$$

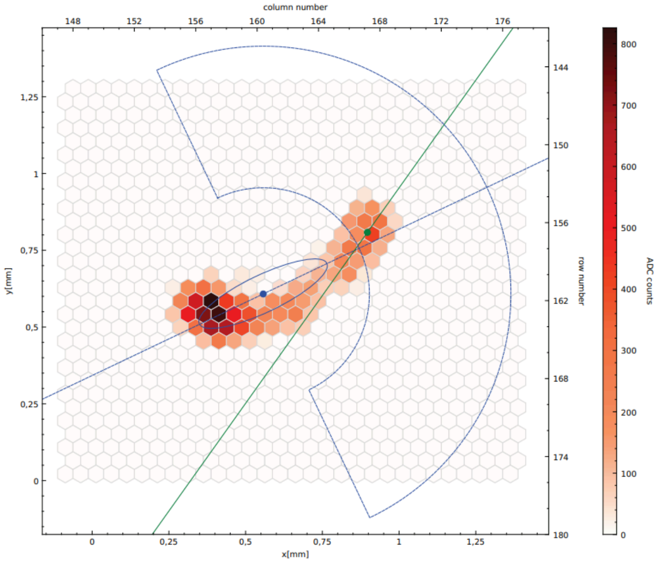
Pol. fraction

EVPA

Step 1: Estimate individual event parameters

Previous/Standard Method:

- Weighted track moments
- Linear energy prediction



$$\begin{bmatrix} \hat{\theta} \\ \hat{x} \\ \hat{y} \\ \hat{E} \end{bmatrix}$$

Soffitta et al. 2021, Astronomical Journal, 2108.00284

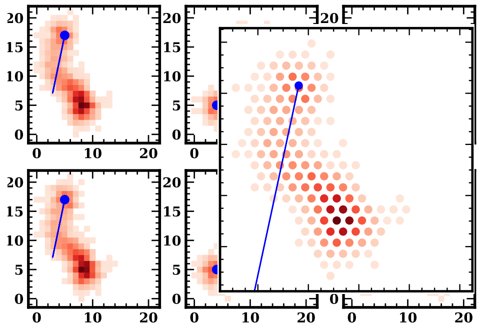
Step 1: Estimate individual event parameters

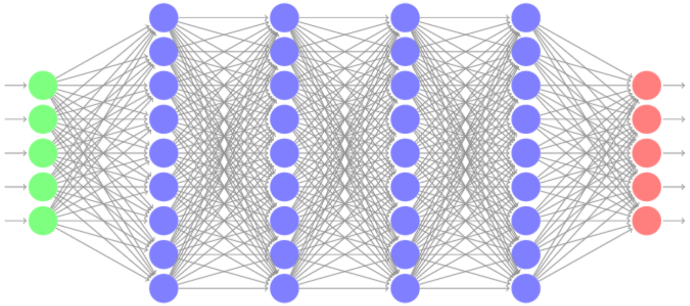
Previous/Standard Method:

- Weighted track moments
- Linear energy prediction

Our Method:

- Convolutional Neural Networks
- Trained on simulated events





$$\begin{bmatrix} \hat{\theta} \\ \hat{x} \\ \hat{y} \\ \hat{E} \end{bmatrix}$$

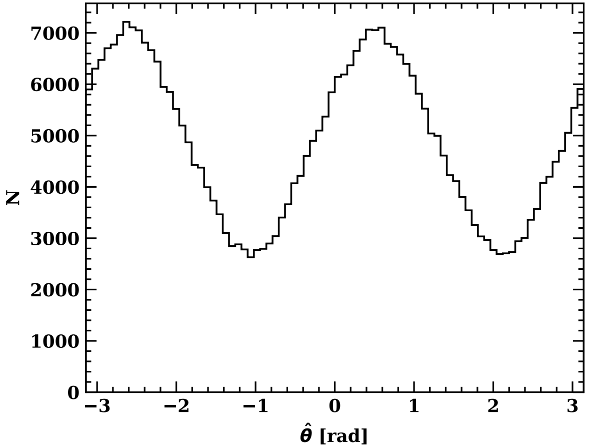
Peirson et al. 2021, NIMA, 986, 164740

NN image - arxiv:1709.02802

Step 2: Estimate polarization parameters

Ensemble of estimated photoelectron angles $\{\hat{\theta}_i\}_{i=1}^{N}$

$$\theta \sim \frac{1}{2\pi}\left(1 + p_0\mu_{100}\cos[2(\theta - \theta_0)]\right)$$



$$I = \sum_{i=1}^{N} 1 = N$$

$$\hat{\mathcal{Q}} = \sum_{i=1}^{N} 2\cos 2\hat{\theta}_i$$

$$\hat{\mathcal{U}} = \sum_{i=1}^{N} 2\sin 2\hat{\theta}_i$$

$$\hat{p}_0 = \frac{1}{\mu_{100}}\sqrt{\hat{\mathcal{Q}}^2 + \hat{\mathcal{U}}^2}$$

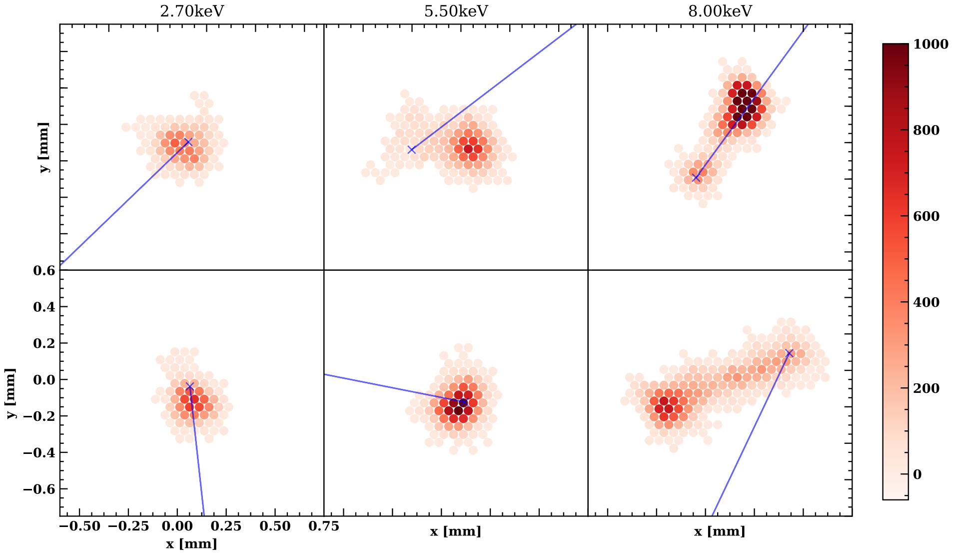
$$\hat{\theta}_0 = \frac{1}{2}\arctan\frac{\hat{\mathcal{U}}}{\hat{\mathcal{Q}}}$$

Kislat et al. 2015, Astroparticle Physics, 68, 45

Step 1+2 works well but we can do better.

Individual Event Weights

- Track direction estimates highly heteroskedastic.
- An event weighted analysis can significantly improve signal recovery.
- But what weights to use?



$$\hat{I} = \sum_{i=1}^{N} w_i$$

$$\hat{\mathcal{Q}} = \frac{1}{\hat{I}} \sum_{i=1}^{N} 2 w_i \cos 2\hat{\theta}_i$$

$$\hat{\mathcal{U}} = \frac{1}{\hat{I}} \sum_{i=1}^{N} 2 w_i \sin 2\hat{\theta}_i$$

$$W_2 = \sum_{i=1}^{N} w_i^2$$

$$\hat{p}_0 = \frac{1}{\mu_{100}} \sqrt{\hat{\mathcal{Q}}^2 + \hat{\mathcal{U}}^2}$$

$$\hat{\theta}_0 = \frac{1}{2} \arctan \frac{\hat{\mathcal{U}}}{\hat{\mathcal{Q}}}$$

$$N_{\text{eff}} = \sqrt{\hat{I}^2 / W_2}$$

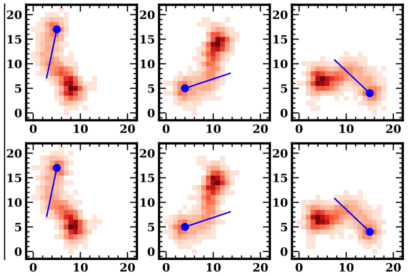
Step 1 Improvement: Deep Ensembles

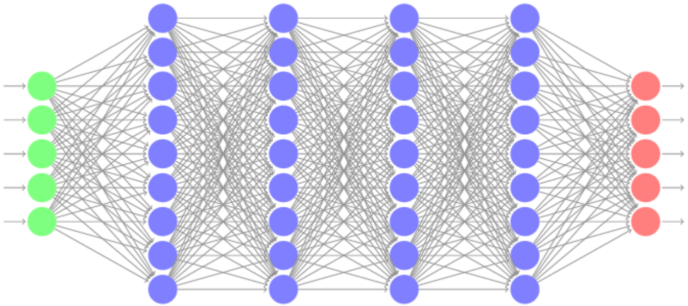
- Quantifying uncertainty on individual event predictions.

$$L \propto \|\hat{\mathbf{y}} - \mathbf{y}\|^2$$

$$L \propto \frac{\|\hat{\mathbf{y}} - \mathbf{y}\|^2}{2\hat{\sigma}^2} + \log(\hat{\sigma})$$

$$L \propto -\hat{\kappa}(\hat{\mathbf{y}} \cdot \mathbf{y}) + \log I_0(\hat{\kappa})$$





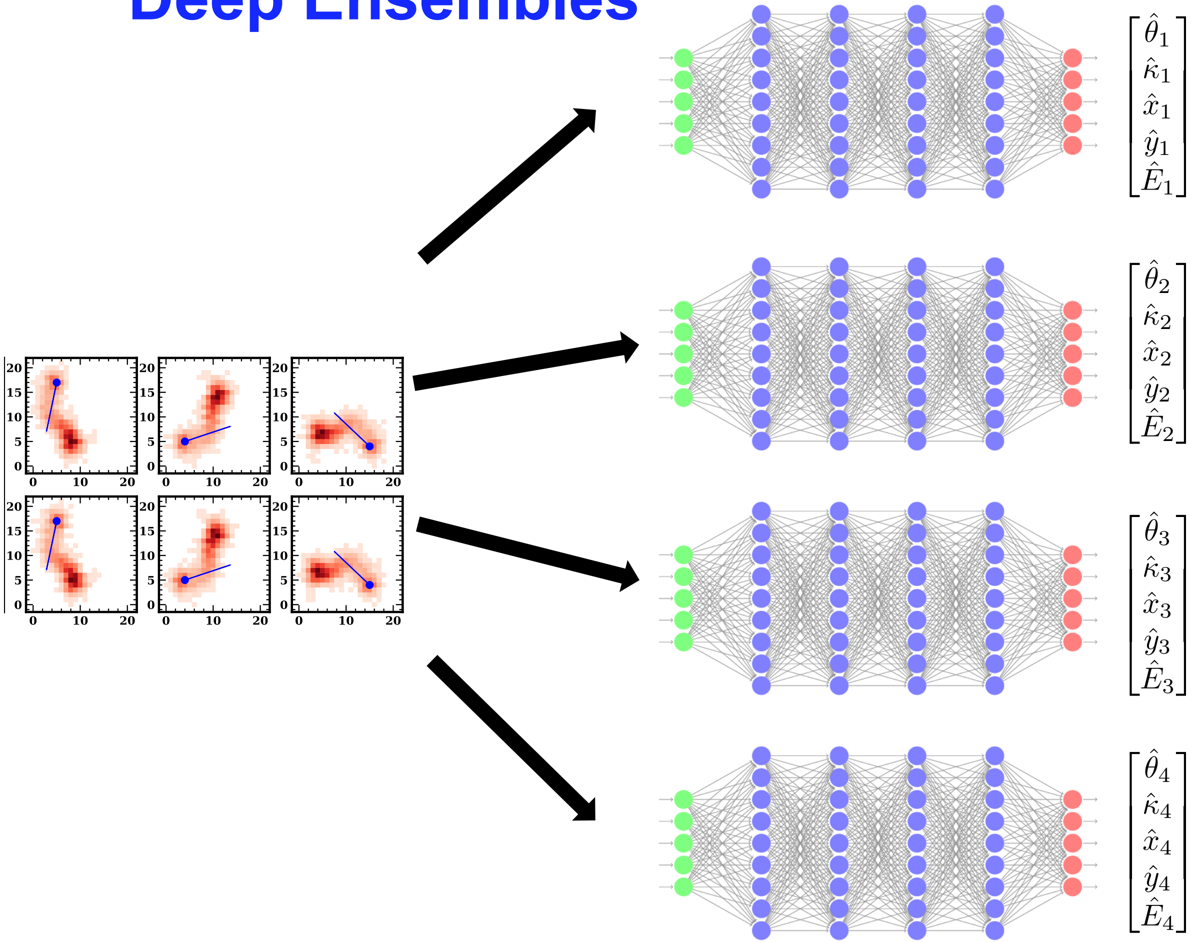
$$\begin{bmatrix} \hat{\theta} \\ \hat{\kappa} \\ \hat{x} \\ \hat{y} \\ \hat{E} \end{bmatrix}$$

Kendall & Gal 2017, NeurIPS 30, pages 5574–5584
Lakshminarayanan et al. 2017, NeurIPS 30, pages 6405–6416

von Mises distribution

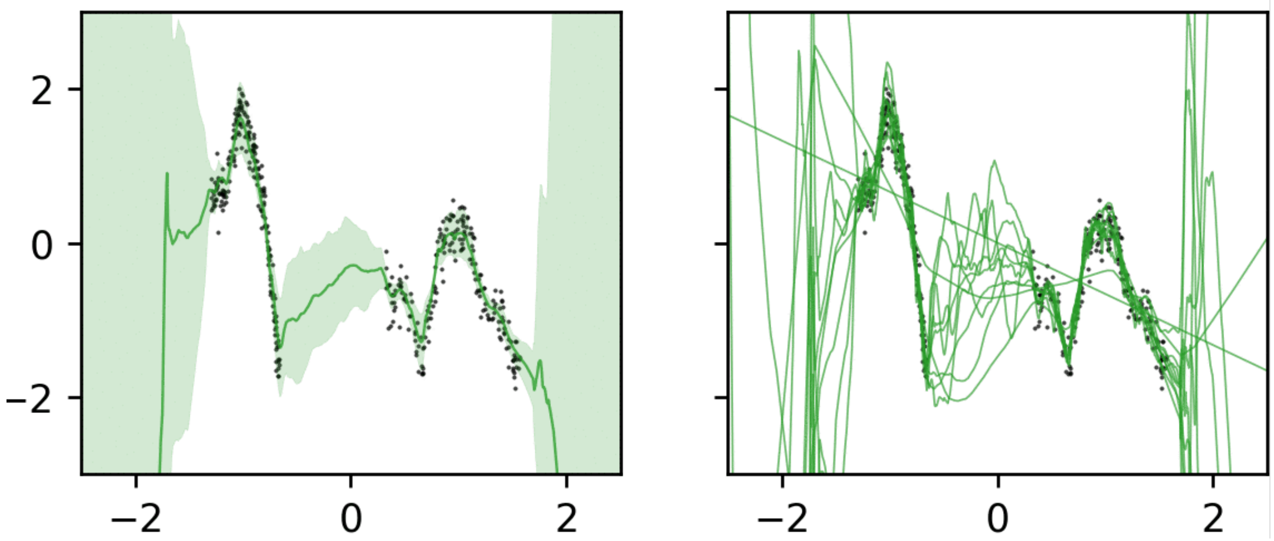
$$\hat{\theta} \sim \frac{1}{2\pi I_0(\kappa)} \exp[\kappa \cos 2(\hat{\theta} - \theta)] \qquad \kappa \sim \frac{1}{\sigma^2}$$

Deep Ensembles



$$\{\hat{\theta}, \hat{\kappa}, \hat{x}, \hat{y}, \hat{E}\}$$

Deep Ensembles



Antoran et al. 2020, NeurIPS, 2006.08437

$$\{\hat{\theta}, \hat{\kappa}, \hat{x}, \hat{y}, \hat{E}\}$$

Step 2 Improvement: Optimal Event Weights

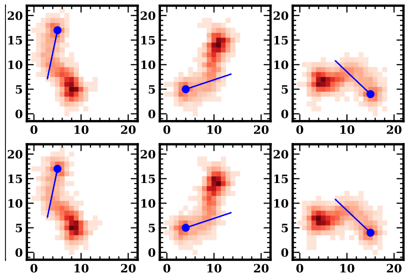
- Translating uncertainties into individual event weights.
- NN predicts $(\hat{\theta}_i, \hat{\kappa}_i)$

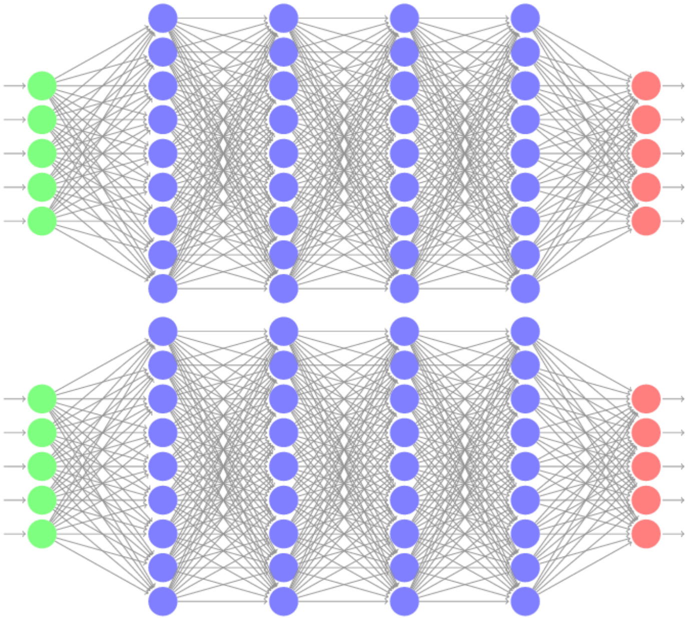
The expected modulation factor (instrument response) for each event gives the optimal weighting to maximize signal-to-noise ratio of the recovered polarization.

$$w_i = \mu_i$$

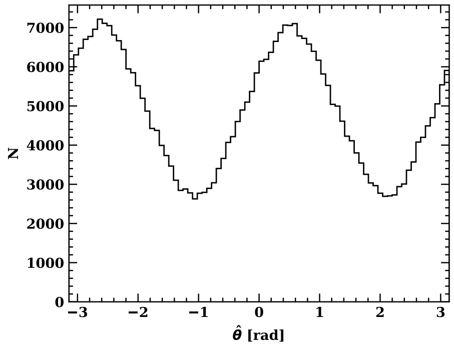
$$\hat{\theta}_i = \theta_i + \epsilon_i \longrightarrow \epsilon_i \sim \frac{1}{2\pi I_0(\kappa_i)} \exp(\kappa_i \cos 2\epsilon_i) \longrightarrow \boxed{w_i = \mu_i(\hat{\kappa}_i) = \frac{I_1(\hat{\kappa}_i)}{I_0(\hat{\kappa}_i)}}$$

Step 1 + Step 2





$$\begin{bmatrix} \hat{\theta} \\ \hat{\kappa} \\ \hat{x} \\ \hat{y} \\ \hat{E} \end{bmatrix}$$



$$w_i = \mu_i(\hat{\kappa}_i) = \frac{I_1(\hat{\kappa}_i)}{I_0(\hat{\kappa}_i)}$$

$$\hat{I} = \sum_{i=1}^{N} w_i$$

$$\hat{\mathcal{Q}} = \frac{1}{\hat{I}} \sum_{i=1}^{N} 2w_i \cos 2\hat{\theta}_i$$

$$\hat{\mathcal{U}} = \frac{1}{\hat{I}} \sum_{i=1}^{N} 2w_i \sin 2\hat{\theta}_i$$

Optimal assuming

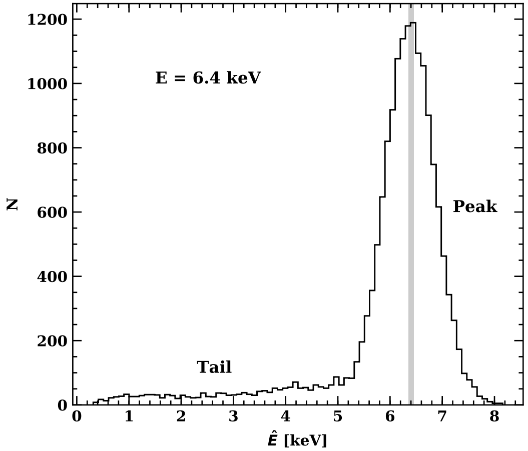
- Ensembles of CNNs are the best angle and uncertainty estimators (almost certainly)
- MLE for Q and U converges to the weighted Stokes estimators (it does)

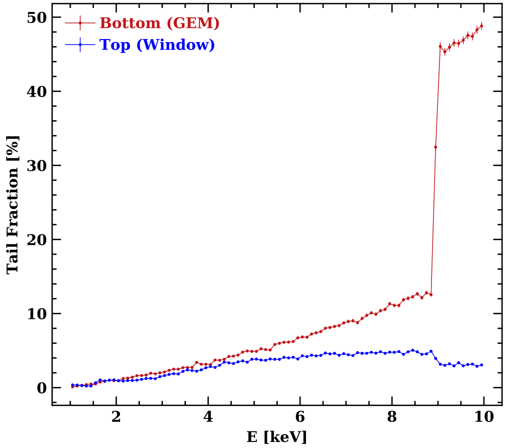
Problem: Tail and Peak Events

Tail = Population of events converting outside of the main detector gas

Peak = Events converting in the main detector gas

- Two types of tail population, top (Be Window) and bottom (GEM).
- Highly energy dependent.





Problem: Tail and Peak Events

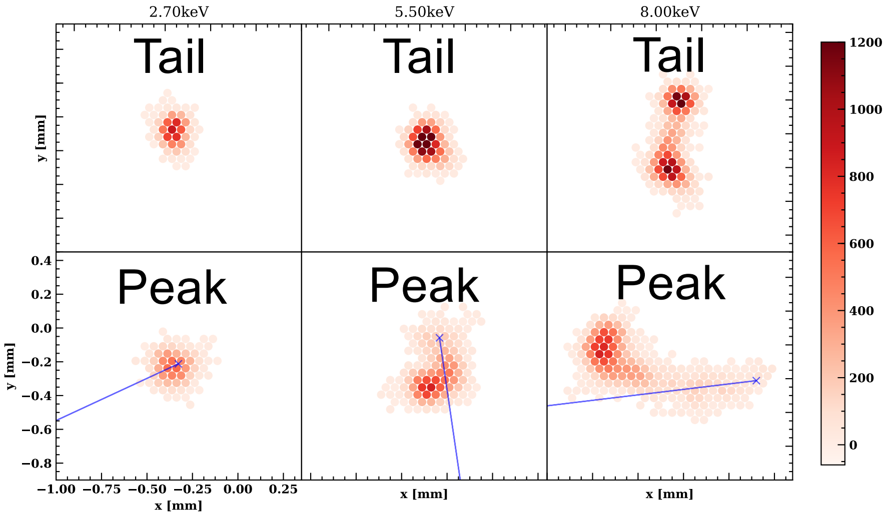
Lots of problems

- Degraded polarization sensitivity (tail $\mu = 6\%$, peak $\mu = 70\%$)
- Degraded energy resolution
- Modulation factor calibration no longer spectrally independent

In theory NNs should be able to weight these tracks appropriately

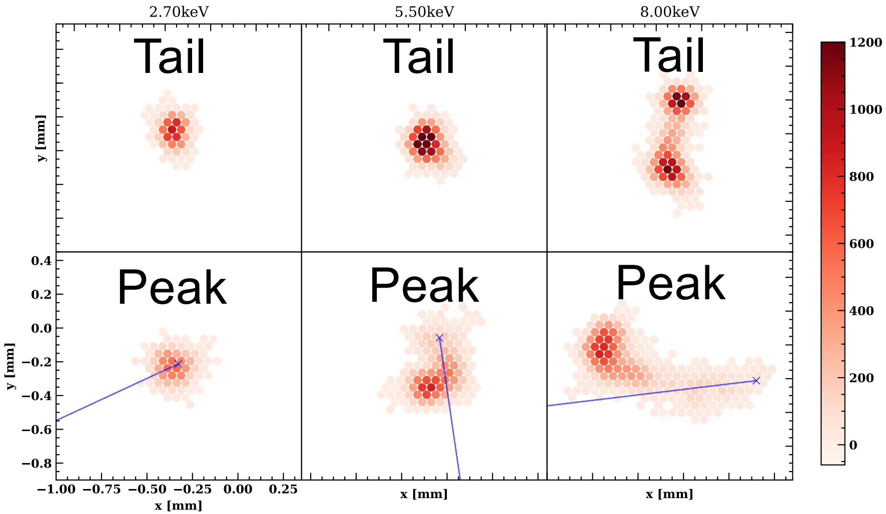
- Unfortunately not the case in practice.
- Weights assigned to tail events are imperfect.

Solution: Identify and remove tail events

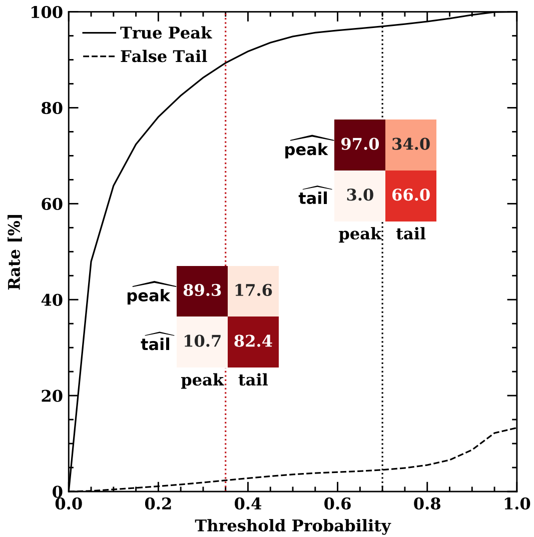


Tail events are structurally distinct from peak events.

Solution: Identify and remove tail events



Tail events are structurally distinct from peak events.



Tail vs. Peak classifier

- Predicts probability of tail for each event.

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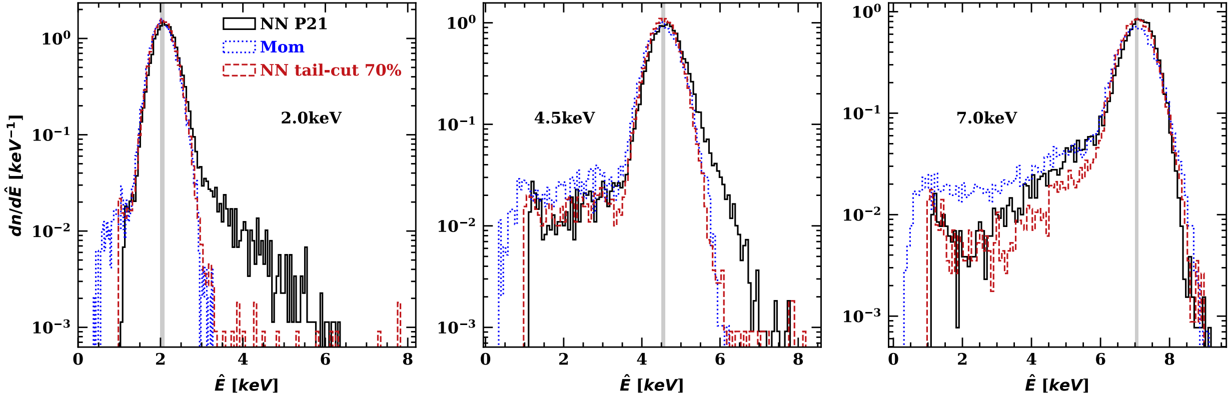
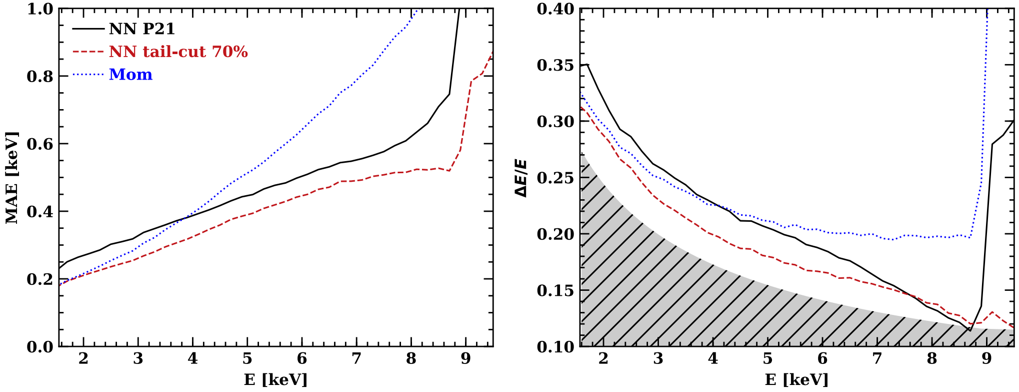
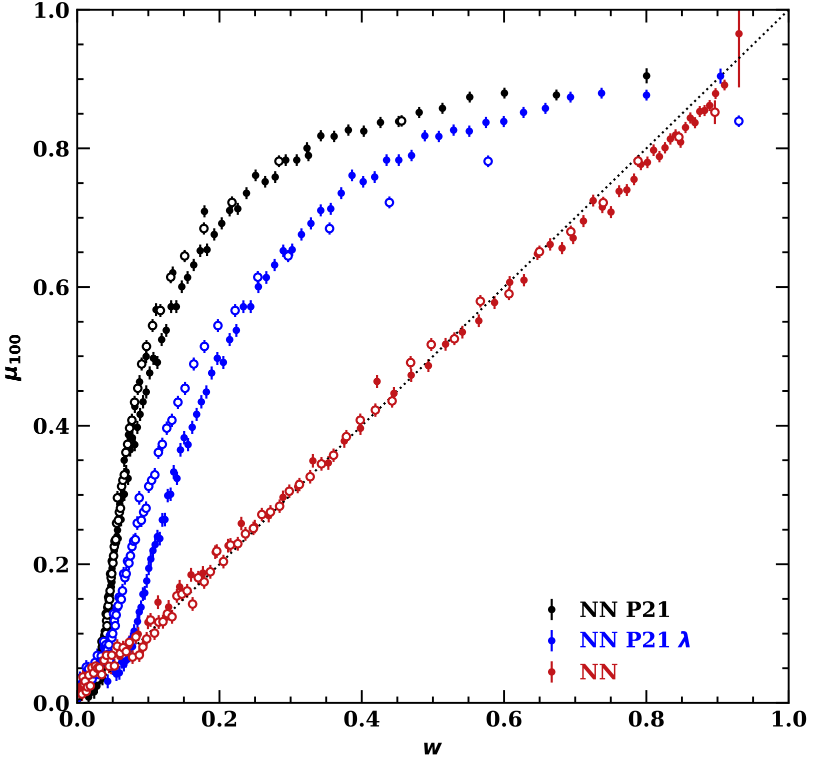
Results: Energy Resolution





Results: MDP99



Modulation factor

Event weight

Spectrum	Method	MDP$_{99}$(%)
PL2	Mom.	5.03 ± 0.02
	Mom. w/ Ellip. weights	4.61 ± 0.02
	NN w/ P21 λ weights	4.09 ± 0.02
	NN w/ opt. wts.	3.75 ± 0.02 ←
PL1	Mom.	4.55 ± 0.02
	Mom. w/ Ellip. weights	4.15 ± 0.02
	NN w/ P21 λ weights	3.65 ± 0.02
	NN w/ opt. wts.	3.38 ± 0.01 ←

Same S/N as weighted moments in 35% less exposure !

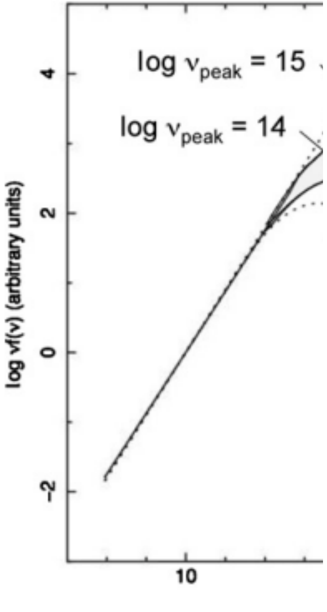
- Weights are expected track modulation factor.
- Spectrum independent, universal weights.

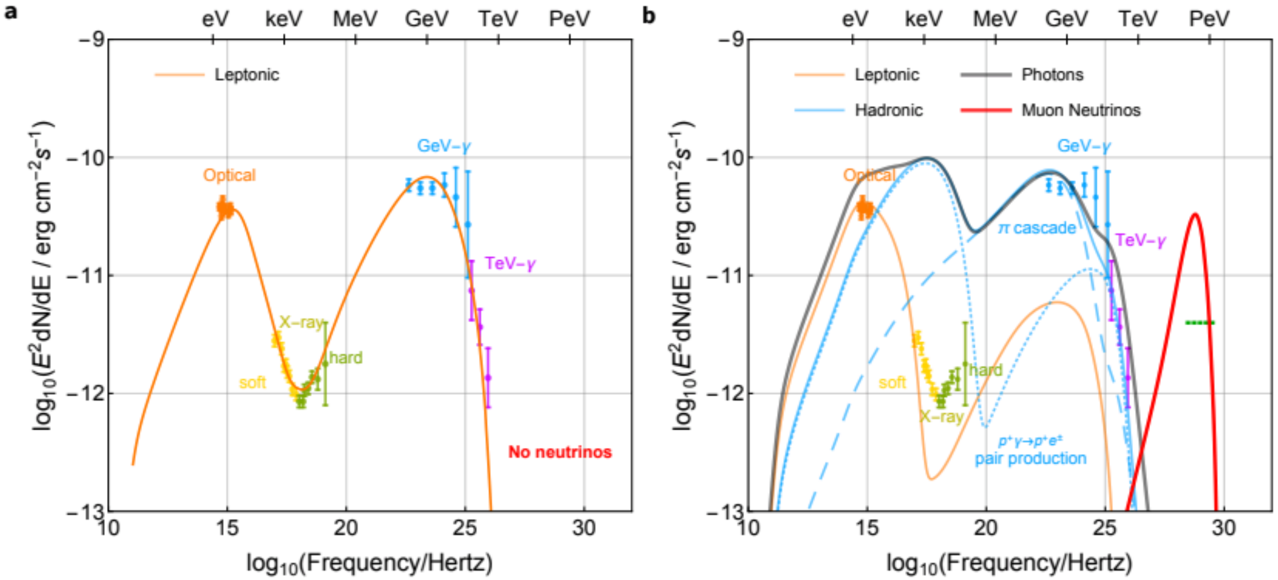
Signal Extraction Summary

- We have derived and applied an optimal weighting scheme for polarization recovery; it's not really possible to do better.
- State-of-the-art MDP and signal-to-noise ratio.
- Tail vs. Peak classifier can be used for both Moment Analysis and NNs.
- We are working together with IXPE calibration team to calibrate and verify this work on flight DUs.

IXPE Application: Blazars

Where does X-ray emission come from?

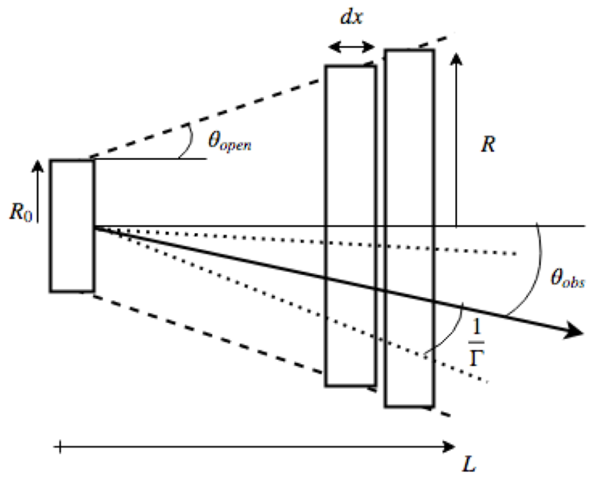


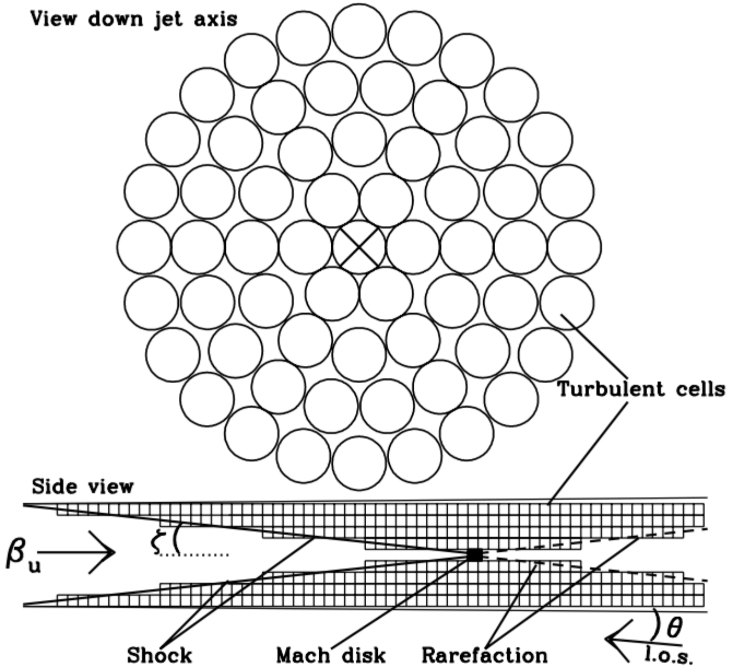


Gao et al. 2019, 1807.04275

Abdo et al. 2010, ApJ 716 30,

Jet Model





TEMZ, Marscher, A. P. 2014, ApJ, 780, 87

- Conical ballistic jet initially in equipartition
- Power law electron population with exponential cutoff
- isotropic pitch angles
- Optically thin

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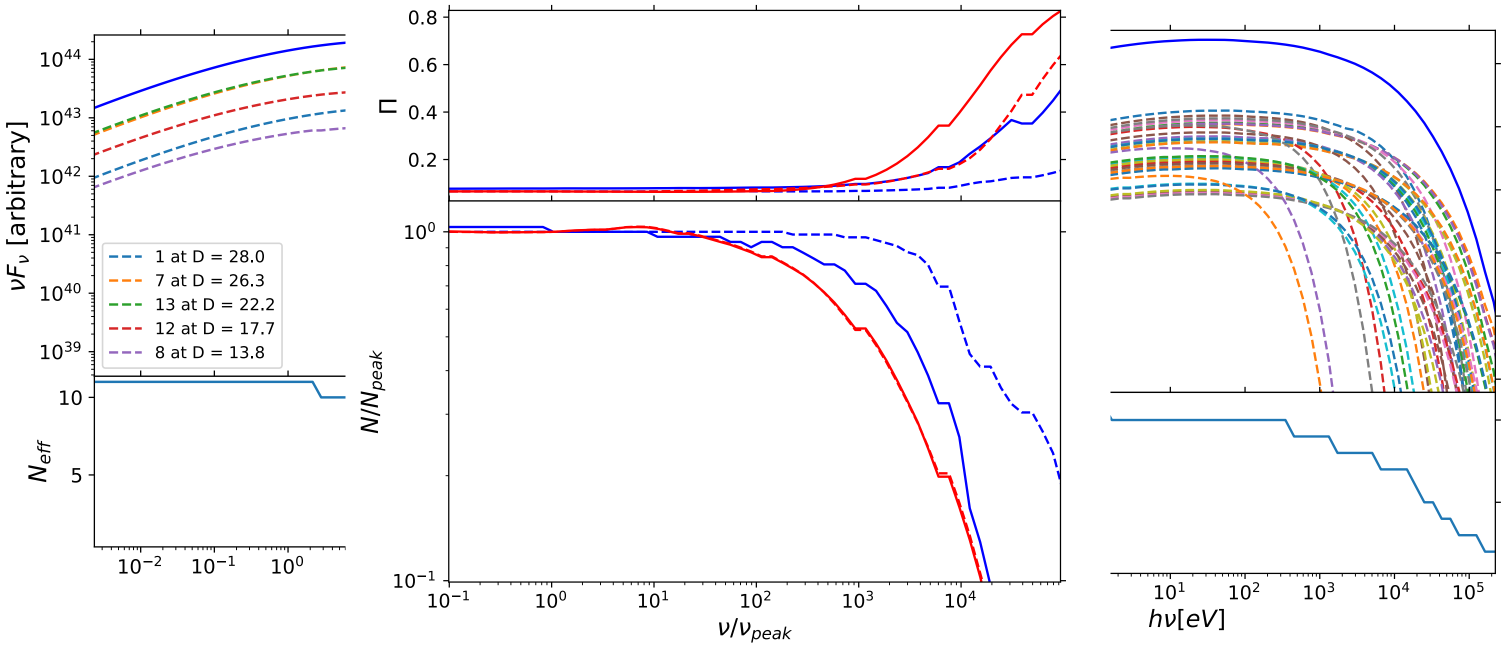
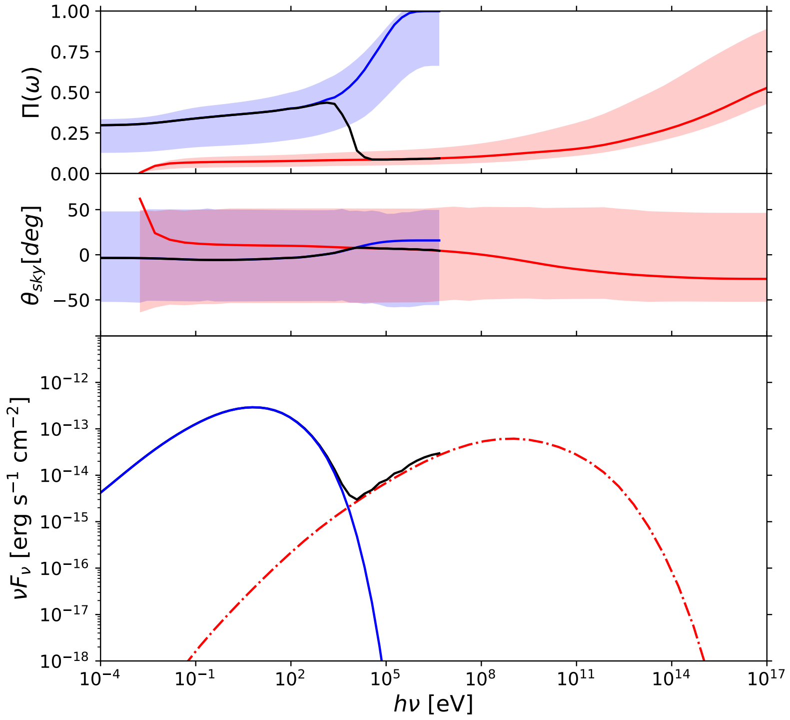
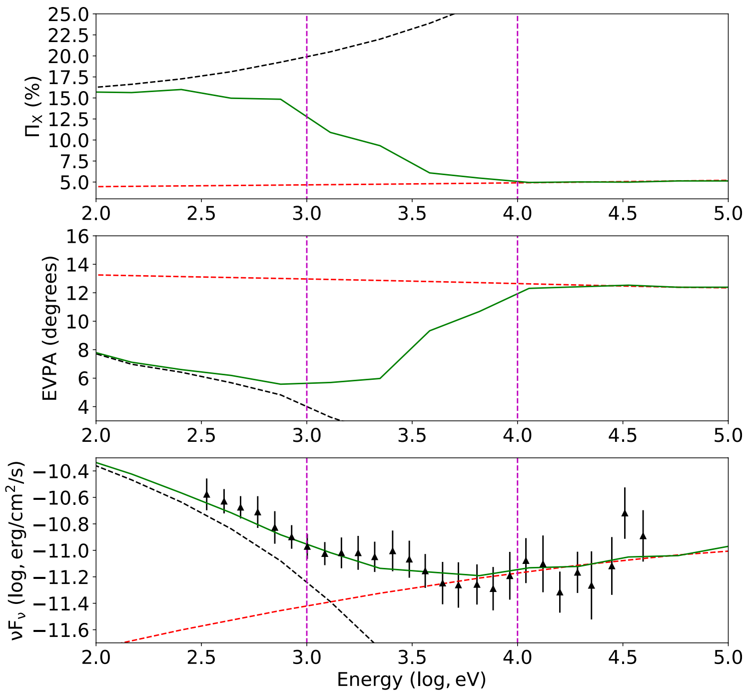
Polarization Predictions

Rising polarization fraction on the synchrotron cutoff



Synchro-Compton Transition





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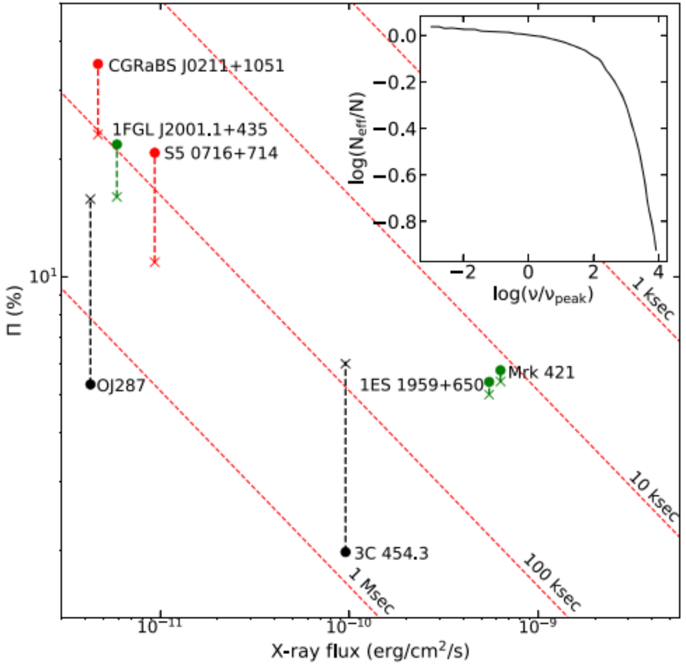
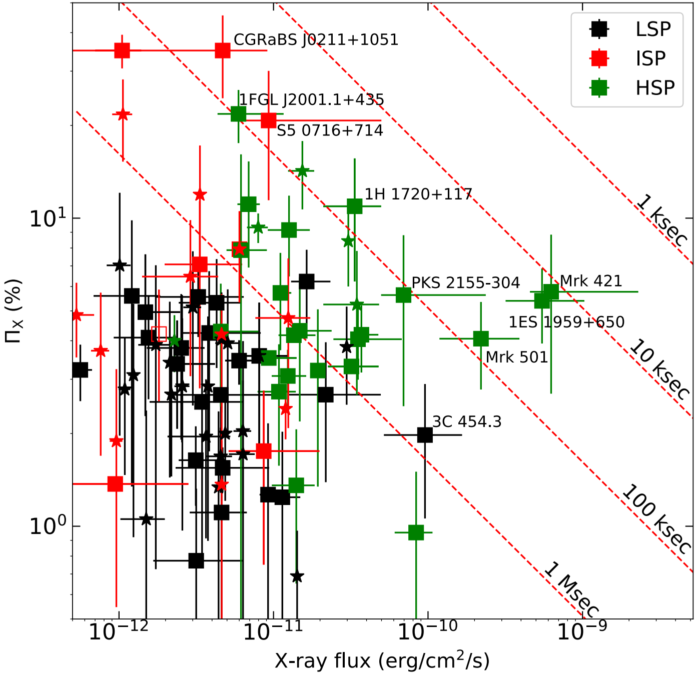
Potential IXPE Targets

Final targets selected are… (can I say this Roger?)

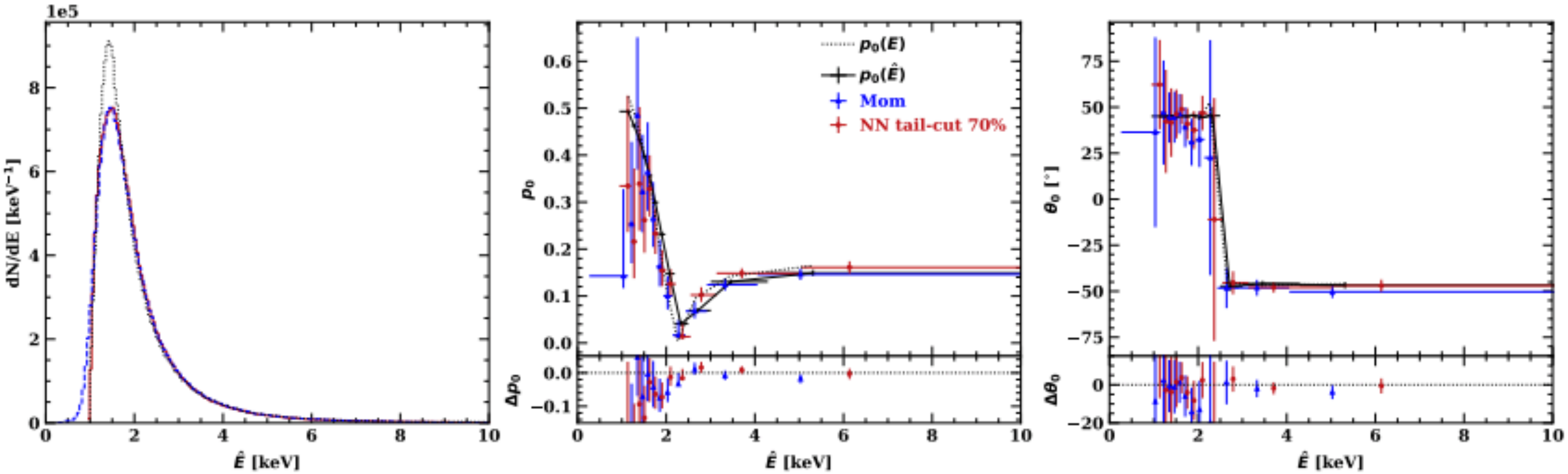




I Liodakis, A L. Peirson, and R W. Romani, ApJ, 880, 1

IXPE Simulation

- 10^{-10} ergs/cm^2/s 2 - 8 keV Source with a 10 day IXPE exposure.



Summary

- X-ray polarization is a powerful probe of jet emission processes and structure.
- Expect increased polarization fraction compared to the peak on a synchrotron cutoff (best for ISPs).
- Expect low polarization fraction for SSC dominated blazar emission.
- With IXPE's improved sensitivity we will soon be able to address some longstanding questions about blazars.